

Quantitative schlieren measurements in a supersonic turbulent jet

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This paper describes the results of a series of measurements made using a single beam schlieren system to investigate the density fluctuations present in the initial region of a supersonic axisymmetric turbulent jet with a Mach number of 1.82 in the flow at the nozzle exit. A preheater was used to reduce the difference between the jet static temperature and that of the surrounding air to a relatively low level. The results show that significant density fluctuations are present in the potential core of the jet and that the distribution of fluctuating intensity across the shear layer differs from that obtained with a subsonic jet without preheating.

1. Introduction

A number of authors have recently reported results of measurements made of the turbulent mixing region of open jet flows using various optical techniques to detect the turbulent flow. Fisher & Krause (1967) used a cross-beam system in conjunction with an absorption detection method based on the injection of a water mist into the jet flow and reported measurements of the convection velocity of the flow as observed by means of the turbulent mixing of the water mist. Subsequently this crossed-beam technique was extended by Fisher & Johnston (1970) to measure the flow in a supersonic jet. More recently, measurements in a subsonic turbulent jet using a crossed-beam system operating on the schlieren principle have been reported by Wilson & Damkevala (1970), who determined the convection velocity in the flow and the intensity of the density fluctuations at points in the flow. The convection velocities in a supersonic jet have been measured by a similar laser schlieren method by Funk & Johnston (1970).

The author (Davis 1971) reported a series of schlieren measurements made in a cold subsonic turbulent jet with a single-beam system. Both axial and transverse component fluctuations of the beam deflexion were measured and it was found that the measured function, the product of the local intensity of the density gradient fluctuations and their integral scale in a plane normal to the jet axis, showed differing distributions through the shear layer for the two components. The difference was ascribed to the effect of the large transverse mean density gradient across the shear layer around the potential core in the initial region of the jet. It was also found that the transverse component fluctuations contained considerably stronger low frequency components.

Single- and cross-beam measurements have also been discussed by Roe (1970) who placed a masking grid across the entire field of view and then measured the unsteady fluctuations passing through the whole aperture. This arrangement has the characteristic of filtering the observed fluctuations directly with regard to the spatial wave-numbers. From this work, as well as of other investigators such as Becker, Hottel & Williams (1967), who used sideways scattering from tracer particles to achieve a limited spatial resolution, it is evident that considerable scope exists for innovation in the mode of application of any particular detection technique in order to achieve spatial resolution, filtering or volume integration directly.

2. Experimental apparatus

A series of experiments has been carried out using a supersonic preheated open jet discharging to the atmosphere. The jet forms part of the test facilities at the Aeronautical Research Laboratories, Melbourne. A detailed study of its operating characteristics has been carried out by Quigg & Catchpole (1956) who determined the Mach number distribution in the core of the jet. With an air inlet stagnation temperature of 200 °C in the upstream settling chamber it was found that the local Mach number in the core was $1.82 \pm 2\%$ at positions 0.35 and 0.64 diameters from the exit plane of the nozzle. Under these conditions the static temperature in the core of the jet would be 12 °C. During the course of the experiments the ambient air temperature was approximately 27 °C, although this might have been slightly modified in the entrained air flow adjacent to the jet shear layer by recirculation of the air discharged from the nozzle. The resulting temperature difference across the shear layer was approximately $\frac{1}{3}$ of the temperature difference which would have been obtained without preheating. Schlieren photographs of the flow from the nozzle, which was 8.68 cm in diameter at its exit, show some evidence of disturbances propagating at the Mach angle (33°) across the exit flow which apparently originate inside the nozzle and would be set up by imperfections in the profile and wall smoothness. However, it appears from the photographs (figure 1, plate 1) that these disturbances do not visibly reflect from the shear layer and therefore only extend to a distance 1.5 diameters from the nozzle exit. In any case, the results of the nozzle calibrations showed that the resulting irregularities in the core Mach number distribution were small. Figure 1(a) also shows that the density in the core of the jet is approximately equal to that of the surrounding air, since there is no overall variation in the light intensity across the jet like that evident in photographs of cold jets with the schlieren knife edge parallel to the jet axis (e.g. Davis 1971).

Fluctuations in the light level at the image plane were measured by means of a photomultiplier tube mounted on a traverse mechanism so as to move normal to the jet axis in the field of view. The photodetector had a small open aperture so that it effectively averaged the instantaneous light level corresponding to a thin beam through the flow of 0.2 cm diameter, this being only 2.3% of the jet diameter. The angular sensitivity of the system was determined by calibrating the photomultiplier signal as the knife edge was moved across the image of the

rectangular source. A remote control motorized traverse was used to adjust the knife-edge position at a rate of approximately 4×10^{-6} m/sec. Similar calibrations have been carried out by Wilson & Damkevala (1970) and the author (Davis 1971). A typical calibration is shown in figure 2, which shows that the use of a rectangular source gives the system a linear response range nearly equal to the full range of light intensity from zero to total cut-off. Some slight irregularities are observed owing to non-uniform illumination of the slit by the lamp

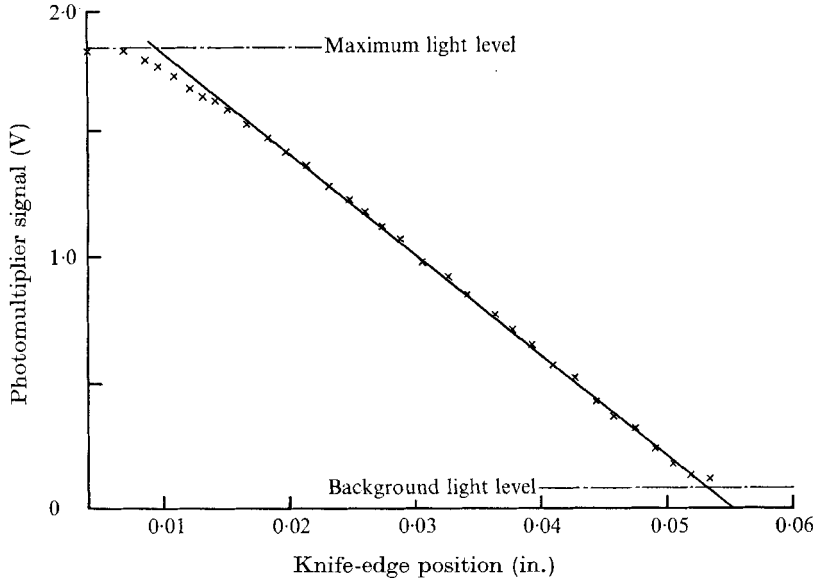


FIGURE 2. Calibration of schlieren photodetector signal by knife-edge traverse.

and collimating lens, although these could be minimized by appropriate adjustment of the lamp image formed on the reverse side of the slit. In particular it was found advantageous to arrange the lamp, which had approximately eight filament coils set close together, so that the coils were at right angles to the plane of the cut-off knife edge. If the lamp was oriented with the coils parallel to the knife edge it was found that periodic fluctuations in the calibration resulted, corresponding to the deflexion of each filament image in turn across the knife edge. A slit size of approximately 0.13 cm at right angles to the knife edge was found to be adequate to accommodate the maximum deflexions due to the test flow.

The fluctuating beam angular deflexion from its mean position is given by

$$\theta' = \frac{E'}{\bar{E} - \bar{E}_0} \cdot \frac{\delta}{2f'} \quad (1)$$

where E' , \bar{E} and \bar{E}_0 are the output voltages from the photomultiplier representing the fluctuating, mean and background levels respectively. The value of \bar{E} was chosen to lie on the mid-point of the overall range of the photodetector output by trimming the knife-edge position. The value of \bar{E}_0 was found to be non-zero

since it was necessary to operate the system with the laboratory open to discharge the nozzle flow and consequently exposed to indirect daylight. Scattering of light from the optical components which were located in a dusty environment caused a significant light level to enter the photomultiplier with the source completely cut off. The size of the slit is δ , at right angles to the knife edge, and f is the focal length of the second mirror, this being the effective length for beam deflexion since the flow is illuminated by a parallel incident beam. A more detailed discussion of similar schlieren arrangements was given by the author (1971).

3. Interpretation of experimental results

Measurements of the distribution of the intensity of the fluctuating beam deflexion angle were made by traversing the photodetector across the field of view at constant distances of 1.5 and 3.0 diameters from the exit plane of the nozzle, moving outwards from the centre-line of the jet to a point where the contribution of the jet disturbances to the total photodetector output was negligible. It was found that the deposition of significant quantities of dust on the optical components because of the exposed environment of the test facility gave rise to a significant variation in the steady average beam intensity across the field of view without the schlieren knife edge in place. Corrections to the measured fluctuating beam intensities were therefore made by first traversing the field of view without the jet operating and recording the distribution of steady light intensity along the line of the traverse. These results were then used to correct the fluctuating beam intensity measured. This was done on a linear basis, assuming that the fluctuations would be attenuated in proportion to the average beam intensity. The angular sensitivity was measured at the centre-line as described in §2, the light level on the centre-line being taken as a reference level.

The results of making traverses at the two positions with a vertical setting of the knife edge to detect the axial deflexions of the beam are shown in figure 3. The measurements shown have not been corrected for background noise components within the frequency range considered (300 Hz to 40 kHz) due to the source, photodetector and laboratory thermal currents. For this reason the signal level does not reduce to zero outside the flow and it was assumed that the constant intensity level outside the flow region represents the sum of these other effects. They are assumed to be uncorrelated with the fluctuations from the jet, so that the latter may be deduced by subtracting the background noise level directly. Whilst it is possible that some fluctuations due to the disturbances outside the jet shear layer may contribute to the observed background level, no significant variation in this level was observed as the photomultiplier was traversed away from the jet and it is assumed that this background level therefore is caused by system noise not associated with the jet flow. The fluctuations in the uncorrected data due to the irregular distribution of dirt on the optical system may be observed, the corrected and smoothed distribution also being marked in. This latter distribution was used to calculate the distribution of fluctuations in the jet. Since the light level at the jet centre-line has been treated as a reference level,

the uncorrected experimental points lie both above and below the smoothed curve.

The unsteady beam deflexion θ'_n is given by the integrated density gradient in a direction \mathbf{n} normal to the knife edge and the beam path:

$$\theta'_n = A \int_{\text{flow}} \frac{\partial \rho}{\partial n} d\zeta, \quad (2)$$

where ζ is the direction along the beam and A is a constant derived from the expression for the variation of refractive index with density for air and is equal

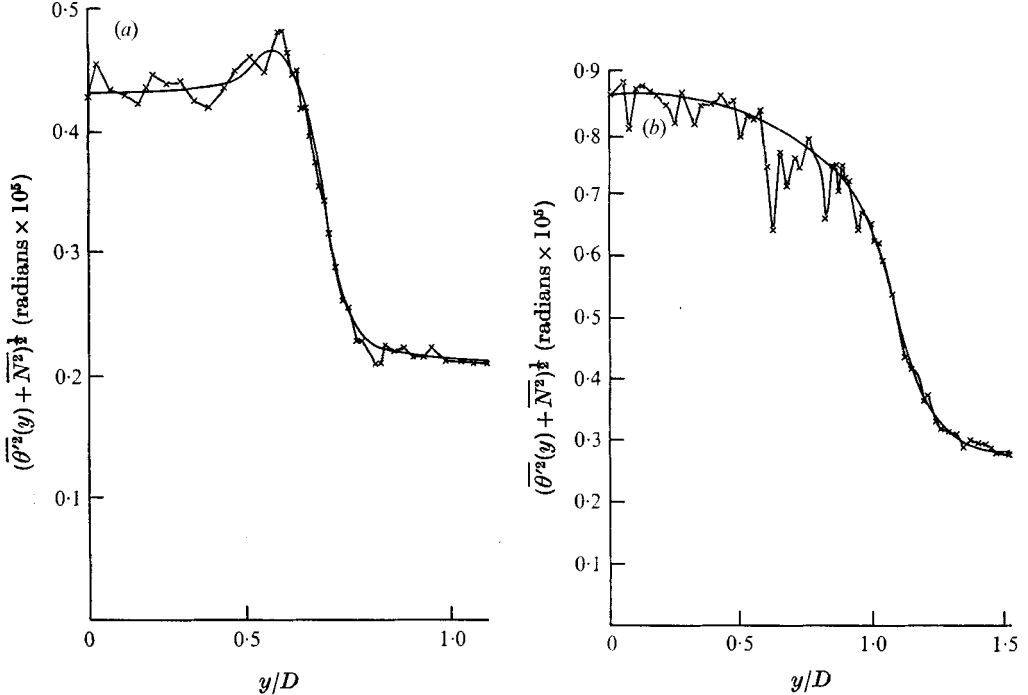


FIGURE 3. Intensity of axial component deflexions of the schlieren beam. (a) $x/D = 1.5$, (b) $x/D = 3.0$, where x is the distance from the nozzle lip and D is the jet diameter.

to $2.37 \times 10^{-5} \text{ m}^3/\text{kg}$. The mean-square intensity of the beam deflexion is then given, for the case where \mathbf{n} is in the axial direction, by the expression

$$\overline{\theta'^2}_a(y) = \frac{2A^2\rho_0^2}{D} \int_y^\infty f_a(r) \frac{r dr}{(r^2 - y^2)^{1/2}}, \quad (3)$$

where y is the normal distance between the measured beam path and the jet centre-line. The jet diameter is D , and the function $f_a(r)$ represents the product of the local intensity of the axial density gradient fluctuations and their integral scale. This function was introduced by the author (Davis 1971) for the axisymmetric jet and is given by

$$f_a(r) = \left(\frac{\partial(\rho/\rho_0)}{\partial(\xi/D)} \right)^2 \cdot \frac{L_\xi}{D}, \quad (4)$$

where ξ is the axial direction co-ordinate and L_ξ is the correlation integral scale in a plane at right angles to the jet axis. It is assumed that this integral scale is the same for all directions in this plane.

Equation (3) is solved numerically for the function $f_a(r)$ from the measured function $\overline{\theta'_a{}^2}(y)$, after correcting the results of figure 3 for the background noise (\overline{N}^2 when represented in terms of its angular equivalent). The results for the two positions are shown in figure 4. Closer to the jet the maximum intensity in the

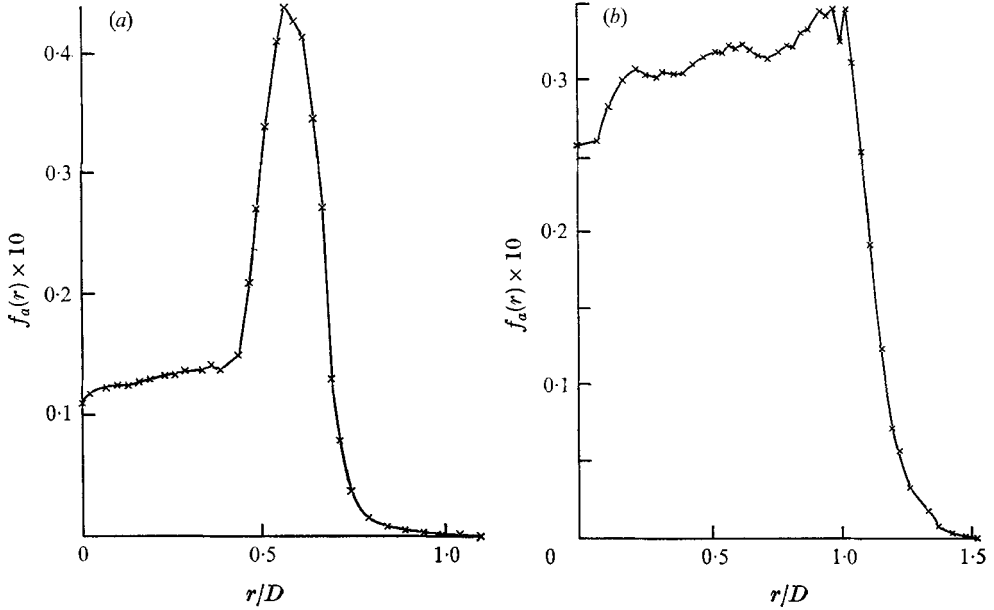


FIGURE 4. Radial distribution of axial component fluctuations across the jet. (a) $x/D = 1.5$, (b) $x/D = 3.0$.

shear layer rises to a sharp peak, with a smaller but significant level of fluctuations across the core of the jet. At the position farther from the jet the shear layer is no longer as well defined and the intensity of the fluctuations in the core of the jet is approximately twice that at the position nearer to the jet. The peak level is, however, reduced.

When the knife edge is set parallel with the jet axis, the schlieren detects the beam transverse angular deflexion θ'_t , which contains components due to both the radial and tangential components of the density gradient. In the absence of a preferred tangential direction the correlation between the two components reduces to zero and the mean-square deflexion of the beam is then given by

$$\overline{\theta'_t{}^2}(y) = \frac{2A^2\rho_0^2}{D} \int_y^\infty \frac{f_r(r) \cdot (y/r)^2 + f_s(r)(1 - (y/r)^2)}{(r^2 - y^2)^{\frac{1}{2}}} r dr. \quad (5)$$

Here $f_r(r)$ and $f_s(r)$ are the products of the intensities of radial and tangential unsteady density gradients and their integral scales (cf. $f_a(r)$, see equation (4)). Since only one radial function may be determined from the single measured

function $\overline{\theta_t^2}(y)$ it is assumed that the two component intensity functions are proportional to one another; that is,

$$f_s(r) = \alpha f_r(r). \quad (6)$$

Solutions for $f_r(r)$ may then be obtained for different values of the constant α . A similar approach was adopted for a series of experiments made previously with a subsonic jet (Davis 1971), the value of the constant α being deduced by finding the value which avoided the introduction of large positive or negative values of the function $f_r(r)$ in the core of the jet.

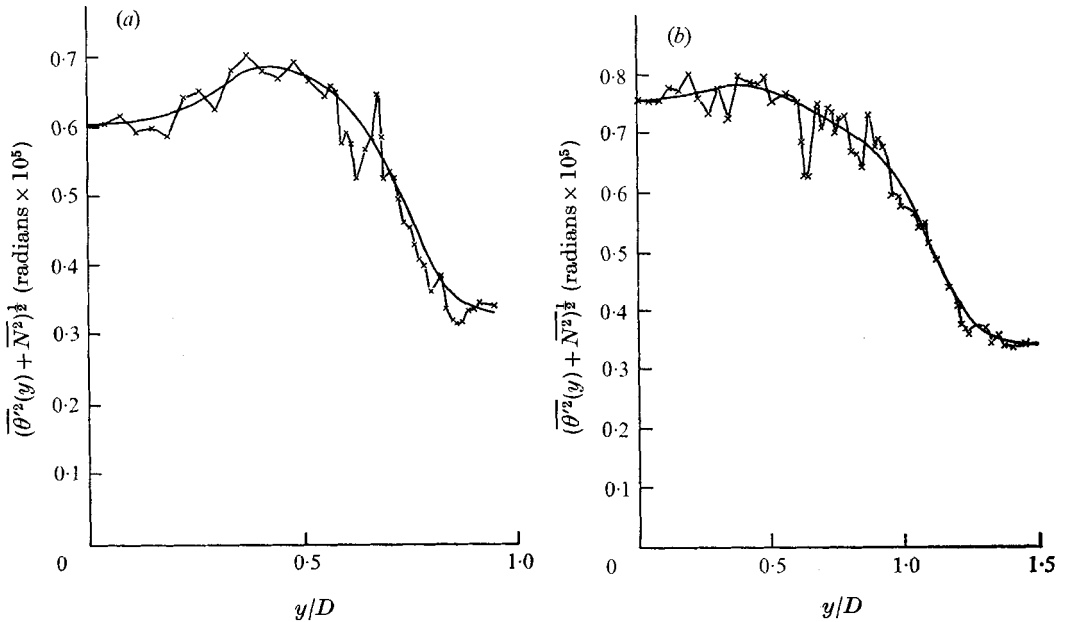


FIGURE 5. Intensity of transverse component deflexions of the schlieren beam.
(a) $x/D = 1.5$, (b) $x/D = 3.0$.

The intensity of the measured fluctuating beam deflexions in the transverse direction is indicated in figure 5, both the experimental data and the curve corrected for beam attenuation by dust being shown. The background noise level may again be observed as the traverse position is moved beyond the jet. The function $f_r(r)$ computed from these results for the same two positions downstream of the jet exit is shown in figure 6. The value of α was again selected so that sharp negative or positive peaks in the function $f_r(r)$ did not arise at the centre of the jet. These peaks, which resulted with different values of α from those shown, were quite sharp, corresponding in width to a region approximately $r/D < 0.05$, and were regarded as being physically unrealistic in the absence of turbulent shear motions in the centre of the jet. A variation of α of $\pm 4\%$ was found to be sufficient to give rise to the peak or dip. It is thus seen that these results indicate that the radial components of density gradient fluctuations are rather stronger than the tangential components, as was also found with the subsonic jet flow.

Frequency spectra of the unsteady beam deflexions were also measured and examples for the two components parallel to and transverse to the jet axis are shown in figures 7 and 8. As has been discussed previously (Davis 1971) the

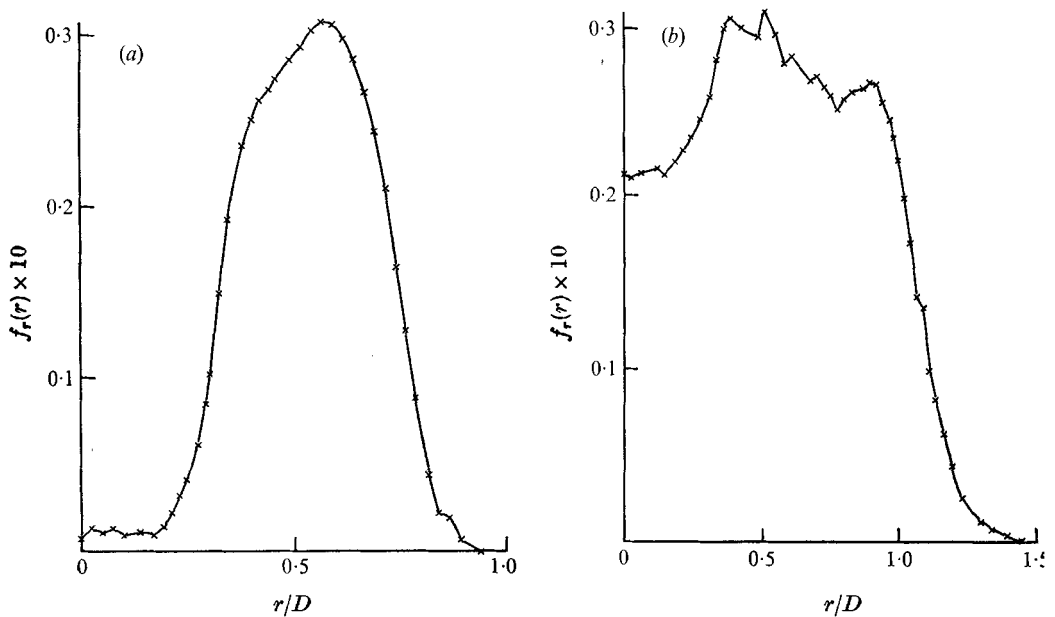


FIGURE 6. Radial distribution of transverse component fluctuations across the jet.
(a) $x/D = 1.5$, $\alpha = 0.94$; (b) $x/D = 3.0$, $\alpha = 0.85$.

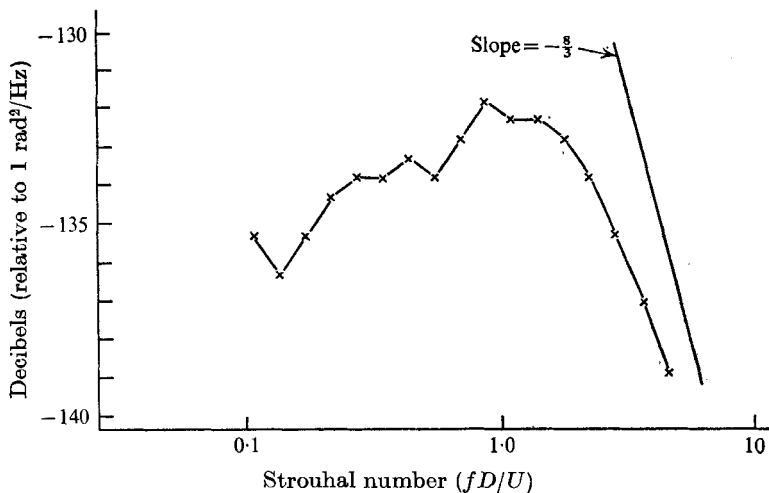


FIGURE 7. Frequency spectrum of axial component beam deflexions.

spectrum of the line-integrated signal differs from that of point fluctuations in the flow. In particular, where the spectrum of the point fluctuations has a negative slope, the spectrum of the integrated signal has a negative slope which is steeper by a factor of κ^{-1} , κ being the wave-number. It may be seen from

figures 7 and 8 that the measured spectra have a lower slope than those reported for the subsonic jet at similar Strouhal numbers. The spectra for the supersonic jet approach a slope of approximately $-\frac{8}{3}$ at the highest Strouhal number considered and give no particular evidence which suggests that there is a portion of the spectrum corresponding to the Kolmogorov relationship. At lower Strouhal

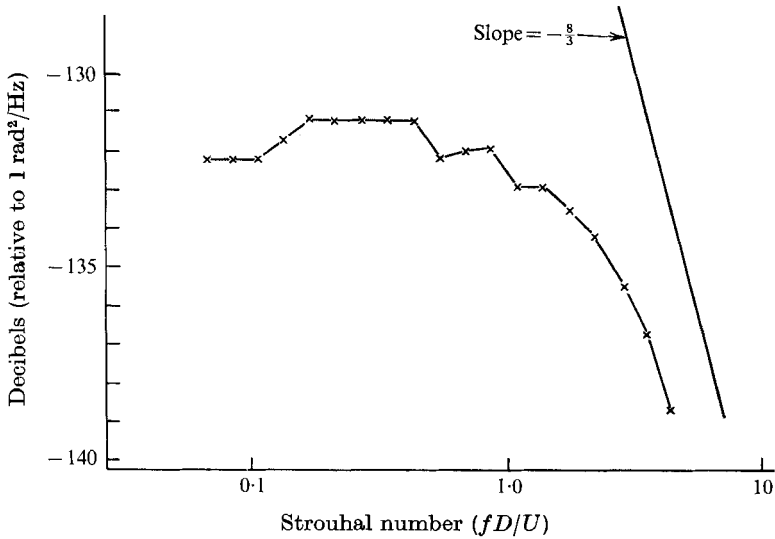


FIGURE 8. Frequency spectrum of transverse component beam deflexions.

numbers the supersonic jet shows a trend similar to that found for the subsonic jet, in that the axial component spectrum is rather more peaked in form, whilst the transverse component fluctuations have stronger low frequency components. It also appears that the location of the peak in the axial component spectrum, at a Strouhal number of approximately 1.0 for the position further from the nozzle exit, is at a rather higher frequency than would correspond to the subsonic data, which for a similar position showed a peak at a Strouhal number of 0.6.

4. Concluding discussion

In considering the results of these experiments in relation to other measurements, particularly those made with subsonic jets having a stagnation temperature equal to ambient temperature, it should be borne in mind that as well as being affected by a supersonic Mach number the results will be influenced by the preheating applied to the flow. The results of the cold subsonic jet measurements (Davis 1971) showed that the relative density fluctuations, taking the average of the estimates given, were 27% at a Mach number equal to 0.9, normalized on the basis of the difference in density between the jet and the surrounding air. Since a variation of the intensity function with the fourth power of the jet Mach number was reported, the density fluctuations would be expected to scale approximately on the difference in density between the jet and surround-

ing air throughout the range of the measurements ($0.3 < M < 0.9$). This conclusion would only be valid if the integral scales do not vary with Mach number. Wilson & Damkevala (1970) reported a similar magnitude for the relative density fluctuations although they found some variation with the jet Mach number, the peak levels being 22% at a jet velocity of 105 m/sec and 34% at 210 m/sec. For both series of experiments the jet was operated with a stagnation temperature equal to that of the ambient air. However, in the absence of exact information regarding the integral scales in the plane normal to the jet axis it is not possible to make a conclusive comparison between the two sets of results.

Since the difference in static temperature between the jet and surroundings for the present experiments is only 15°C the corresponding difference in mean density across the shear layer is reduced to 5% of the ambient air density. Hence the gradients of mean density and temperature across the jet shear layer will be relatively much smaller than for jets operated at comparable Mach numbers without preheating. The peak values of the functions $f_a(r)$ and $f_r(r)$ do, in fact, lie below the extrapolation of the subsonic data (Davis 1971). Davis (1971) indicates peak values of 0.48 and 0.62 for the maximum values of the two functions respectively, whereas the maximum values measured in the supersonic flow were 0.0349 and 0.0304 at $x/D = 3.0$ (x being the distance from the nozzle lip). This large reduction is clearly due to the partial elimination of the Mach number dependent density change across the shear layer.

The effects of maintaining a nearly equal static temperature in the jet and surrounding air are also to be observed in the radial distribution of the intensity function $f_r(r)$. For the cold subsonic jet flow data a double peak in this function was observed, with a maximum to either side of the centre of the shear layer. This effect was ascribed to the turbulent mixing of the cold jet with the surrounding air since the intensity function $f_r(r)$ would be proportional to $|\partial^2 \bar{\rho} / \partial r^2|$ if the process were considered as the mixing of a passive scalar quantity in the shear layer. Owing to the monotonic increase of mean density $\bar{\rho}$ across the shear layer, the second derivative has two maximum modulus values. The present data (figure 6) do not show this same effect and it is concluded that the effects of the density change across the shear layer are relatively small. This conclusion is further supported by schlieren photographs of the two flows with the knife edge parallel to the jet axis. Whilst the cold subsonic jet showed an asymmetric distribution of mean light level about the jet axis in the photograph, this is not observed in the photograph of the supersonic jet (figure 1(a)). However, it may be observed in this photograph that some transverse mean density gradients, which appear as alternate light and dark regions, are present in the shear layer around the jet whilst the body of the jet shows no apparent variation of mean density. It is likely that heat transfer to the nozzle wall has produced this local effect by the introduction of a decrease in the flow temperature in the nozzle boundary layer.

The distribution of both components of the density gradient fluctuations at the position further from the jet exit ($x/D = 3.0$) shows that the local fluctuations are relatively strong right across the core of the jet and also extend into an annular region around the shear layer, as indicated by the distribution of local

Mach number across the jet. The turbulent intensity distributions obtained by Fisher & Johnston (1970) show a much smaller transverse spread, being essentially confined to the turbulent shear layer itself although a significant outward radial displacement of the point of maximum intensity was observed in their experiments. This outward movement increased with distance from the jet exit and would correspond to a peak intensity of mixing at $r/D = 0.68$ at a position three diameters from the jet exit with a Mach number in the core equal to 2.46. This again suggests that the results of the present experiments do not contain strong components due to passive mixing of the jet flow but that the observed fluctuations are mainly caused by the inertially produced turbulent stress fluctuations in the turbulent flow. It appears that the strong density fluctuations in the near field of the sound radiated from the shear layer are also detected by the schlieren system, since the observed fluctuations extend beyond the turbulent mixing region. As these fluctuations would only propagate in the core of the jet within Mach cones originating in the shear layer, it is seen that further support of these conclusions is given by the intensity profiles at the position closer to the jet orifice ($x/D = 1.5$). Here the level of the fluctuations in the core appears very much smaller, especially for the transverse component close to the centre of the core of the jet. The point on the jet centre-line at this station would only be influenced by fluctuations originating within the nozzle itself. From this it may be seen that the axial component distribution at $x/D = 1.5$ indicates that there are fluctuations in the core which are propagating axially downstream from the nozzle itself. Since no particular precautions were taken to eliminate noise from the air supply to the settling chamber upstream of the nozzle, it is possible that this effect is at least contributing to the observed axial component fluctuations in the core of the jet. Severe disturbances of this nature were not expected since the air supply system did not include any choked valves. It is also possible that some disturbances may originate from the nozzle boundary layers. The relatively broader spread of the transverse component fluctuations at this position close to the nozzle exit and their relatively small amplitude at the centre of the exit flow indicates the radiation of disturbances outward from the shear layer in either direction, constrained within the Mach angle in the supersonic core region of the jet. The axial component fluctuations show a strong peak in the shear layer, corresponding in extent approximately to the shear layer thickness, but do not extend significantly outside this region except for the fluctuations found in the core as discussed.

The frequency spectra obtained in the supersonic flow resemble those obtained for the cold subsonic jet in so far as the spectral densities at lower frequencies ($St < 1.0$ approximately) are lower for the axial component than for the transverse component fluctuations. However, this difference does not appear to be as large for the supersonic case and was also found to decrease with increasing Mach number for the subsonic jet. From these results it is not clear to what extent the stronger low frequency transverse components may be associated with changes in Mach number and temperature difference between the jet and the surrounding air, although it does appear that the difference in mean density, which predominates in the subsonic measurements, is significant in this respect, possibly because

of the combined effect of the relatively large transverse mean density gradients and larger-scale disturbances in the flow. The spectra obtained by Fisher & Johnston (1970) from absorption due to water vapour also show a rather broad peak of intensity at a Strouhal number of 1.7 based on the distance from the jet exit plane. The maximum of the single schlieren beam spectral density for the axial component fluctuations is at a Strouhal number of 1.0 (see figure 7) based on the jet exit diameter at a distance three diameters from the exit. From this it may be seen that in spite of the lower Mach number for the schlieren measurements (1.82 compared with 2.46 and 3.34 for the absorption measurements of Fisher & Johnston), the maximum intensity is at a relatively higher Strouhal number for the schlieren beam than for the absorption beam by a factor of rather over two. It may also be noted that the maximum for the cold subsonic schlieren measurements lay at a Strouhal number of 0.6 based on the jet diameter at $x/D = 3.0$, corresponding approximately to the supersonic absorption data. Hence it appears that the density fluctuations due to turbulent stress fluctuations, which predominate in the data presented here owing to the near elimination of the difference in density between the jet and surroundings, have a maximum intensity at a rather higher Strouhal number than the fluctuations due to turbulent mixing. In addition, the single-beam schlieren spectra within the range of the present experiments have negative slopes which are considerably lower than those of the other single-beam spectra for Strouhal numbers greater than unity.

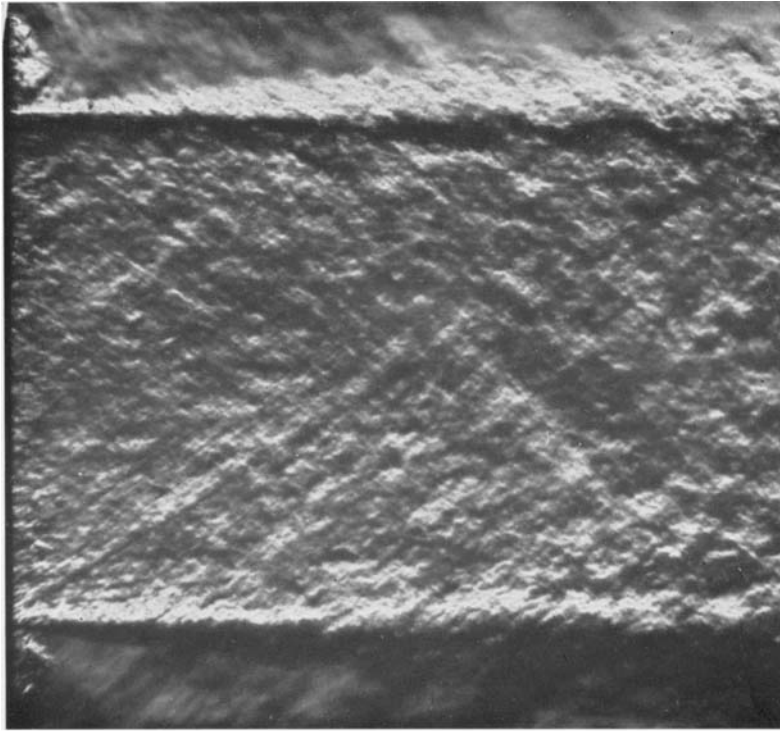
In conclusion, it has been found that the schlieren measurements made with the heated supersonic jet show significant differences when compared with similar measurements for a cold subsonic jet as well as with the supersonic mixing measurements of Fisher & Johnston. The distribution of fluctuating intensity across the shear layer in the present experiments has maxima considerably less than the extrapolation of the subsonic data obtained without preheating. Also, the form of the distribution of the transverse component fluctuations shows that the strong double peak due to the transverse mean density gradient of the subsonic jet flow is absent for the supersonic flow. Both these results indicate that the observed fluctuations for the heated supersonic jet are caused primarily by the turbulent stress fluctuations rather than by turbulent mixing, although it appears likely that some contribution to the measured fluctuations may still exist owing to mean temperature variations in the nozzle boundary layer and to the small difference in temperature between the jet and surroundings. This conclusion is further supported by the existence of strong fluctuations outside the shear layer itself but within the Mach angle in the core of the jet, where the observed fluctuations would be due to pressure disturbances emanating from the shear layer. The frequency spectra show that the density gradient fluctuations have relatively stronger high frequency components in the supersonic case as compared with the other measurements which essentially indicate turbulent mixing. Also, the characteristic Strouhal number is higher. Evidence indicating that the transverse component density gradient fluctuations have stronger low frequency components is again found, although this effect is not so marked for the heated supersonic jet. It should be noted that the measured functions of radius represent

the product of local intensity and integral scale, and that the latter is assumed to be constant for different directions in the plane normal to the jet axis.

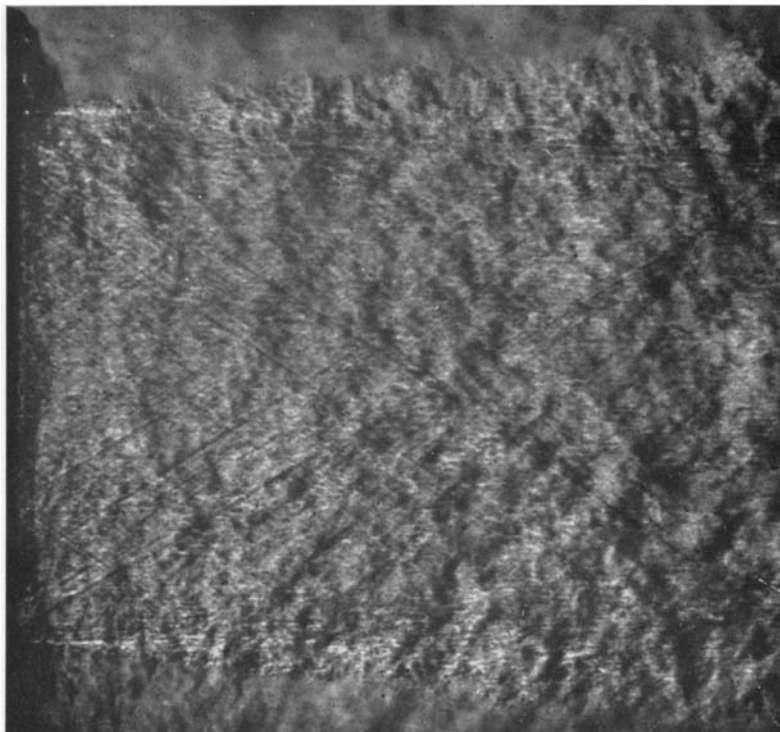
The co-operation of the director and staff of the Aeronautical Research Laboratories, Melbourne, where these experiments were carried out, is gratefully acknowledged. The assistance of Mr M. Culley and Mr D. G. Stewart in particular has made it possible to carry out this work.

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(a)



(b)

FIGURE 1. Instantaneous spark schlieren photographs of $M = 1.82$ jet, (a) knife edge set parallel to jet axis, (b) knife edge set normal to jet axis.

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